Report

Estimation of Hailstone Radii Using the Collatz Conjecture

Abstract

There is an alleged connection between a much researched, yet unsolved math problem, the Collatz conjecture, and a quite common, damaging meteorological phenomenon: hailstorm. The relationship would be that, in sequences generated by Collatz algorithm, the way in which numbers rise and fall resembles hailstones going up and down inside a cloud, whence the name “Hailstone sequences”. The aim of this paper is two-fold: first, to use JavaScript to research on the Collatz conjecture with the perspective of a high-school student. Our algorithm tested a generalized form of the conjecture for multiple primes (3, 5, 7) and signs (+,). The Pearson correlation coefficient found between the initial value and, respectively, the total stopping time or the maximum value reached excluded any linear correlation. The second (and main) goal was to assess the hypothesis whether hailstones could indeed follow a Collatz-like function trajectory, studying the implication on the radii of them. Introducing the concept of *conversion formula,* we estimated the final radii for different functions (straight line, square-root, square, logarithmic, exponential), unit of measures (from Km to mm), and starting heights ranging from 4000m to 10000m, should the motion of hailstones behave like a Collatz function. In all but one case, we did not get radii believable in size, and *reasonably* randomly distributed. For the linear formula (in cm), the -test values between our estimated values and Nelson’s model values are above the critical values. Hence, we should reject the initial hypothesis.

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CREST AWARDS GOLD

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Introduction

The Collatz conjecture is one of the most famous, researched, unsolved problems in mathematics and enjoys the appealing property, often attributed to celebrated number-theoretic questions, of being simple to state and apparently impossible to answer. The problem could be formulated as follows: consider the integer-valued function defined as

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even,} \\ 3n+1 & \text{if } n \text{ is odd,} \end{cases} \qquad (1)$$

where n is any positive integer number. The Collatz conjecture states that, no matter what the starting positive integer n is, the function iterates of $f(n)$ eventually reach the value 1.

- Stated in 1937 by Lothar Collatz, the conjecture has been checked for all values up to 268 and yet no proof has been found as of 2020. “Mathematics may not be ready for such problems” Paul Erdos stated. Despite this pronouncement, the Collatz conjecture keeps attracting the interests of many mathematicians, and the study of the problem has not been without reward.
- There are survey papers on the topic, but they are intended either for high-level mathematicians (Lagarias, 1996) or for undergraduate students (Lohia, 2022). We aim to complement the abstractness of the other papers following the perspective of a high-school or an undergraduate student who approaches this question for the first time. This means discussing naïve approaches and why they do not work, concrete examples, and general Collatz-like functions, using JavaScript algorithms and computations.
- Besides this, we aim to spark interest in real world applications of the Collatz conjecture to adopt different perspectives of this unsolved problem and gain insights from them. Here we address the statement that the sequences of numbers involved in the iteratives of the function $f(n)$ had been called the “Hailstone Sequences” because the values seem to rise and fall multiple times, allegedly as a hailstone inside a cloud. We study at which extension this is an accurate description and what the implications would be if the formation of hailstones indeed followed a Collatz-like function pattern.

Why is the Collatz conjecture so hard?

A lot of different approaches were used to understand better the sequences that Collatz algorithm produces. Dealing with a statement involving positive integers, a natural approach would be to use strong induction: indeed, for the starting number 1 the problem is automatically true, so we need to check the inductive step. If we assume that the Collatz algorithm reaches the value 1 for all the integers less or equal than n, if n is odd then $n+1$ is even and $(n+1)\ /\ 2$ is less or equal than n: so by induction the Collatz sequence starting at $n+1$ converges as well.

Now assume that n is even: then $n+1$ is odd and so $f(n) = 3n+4$ and $f^{(2)}(n) = (3n+4)/2$ which is never less than or equal to n. However, $(3n+4)\ \ /\ 4 < n$ for any integer greater than 4. After checking for $n = 2{,}3{,}4$ that the conjecture still holds, we can conclude that if $f(n)$ is divisible at least by 4, Collatz converges also starting at $n+1$. If it is not the case, then $f^{(2)}(n)$ would be odd and

$$f^{(3)}(n) = 3\left(3\frac{n}{2}+2\right)+1 = 9\frac{n}{2}+7. \qquad (2)$$

More precisely, using the following equivalent way to write the Collatz function $f(n)$

$$T(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even,} \\ \frac{3n+1}{2} & \text{if } n \text{ is odd,} \end{cases} \qquad (3)$$

by induction one can prove (Lagarias, 1996, p. 8) that the $k-th$ iteration $T^{(k)}(n)$ can be expressed as

$$T^{(k)}(n) = \frac{3^{\sum_{i=0}^{k-1} x_i(n)}}{2^k} n + \sum_{i=0}^{k-1} x_i(n) \frac{3^{\sum_{j=i+1}^{k-1} x_j(n)}}{2^{k-i}} \qquad (4)$$

where $x_i(n)$ reflects the parity of the $i-th$ iteration $T^{(i)}(n)$, assuming value 1 if $T^{(i)}(n)$ is odd, and value 0 if $T^{(i)}(n)$ is even. However, at first sight the equation $T^{(k)}(n) < n$ seems hard to solve. To understand at which extension this perception is true, we can try to analyze the behavior of $f^{(k)}(n)$ (and therefore of $T^{(k)}(n)$) via software[1]. We considered *Collatz-like function* of the form

$$f_{p,+}(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even,} \\ pn+1 & \text{if } n \text{ is odd,} \end{cases} \quad \text{or} \quad f_{p,-}(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even,} \\ pn-1 & \text{if } n \text{ is odd,} \end{cases} \qquad (5)$$

With p = 3, 5, or 7 respectively, and for n ranging from 1 to 10000 and for each choice $(p, n, sign)$, we calculated:

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1 See Appendix 1 for a brief explanation of the JavaScript algorithm, as well as the hyperlink to GitHub folder containing the algorithm itself.

a. whether it generates a sequence which reaches 1 (type 1), it forms a cyclic sequence which does not pass by the value 1 (type 2) or if it exceeds the value 2147483647 (it is a Java limitation) (type 3);

b. the number of iterations needed to reach 1 (type 1), called the *total stopping time* (Lagarias, 1996, p. 5) or even form a cyclic sequence which does not pass by value 1 (type 2) and denoted by $\sigma(p, n, \pm)$ for a fixed prime p and the sign, if n is of type 1 or 2;

c. fixed the prime p and the sign, if n is of type 1 or 2, the maximal value reached by the sequence $\{f^{(k)}{}_{p,\pm}(n)\}_{k\geq 0}, denoted\ by\ \max(p, n, \pm)$;

d. fixed the prime p and the sign, the (sample) Pearson correlation coefficient [2] $r_{n,\sigma}$ between the initial number n and the total stopping time,

e. fixed the prime p and the sign, the (sample) Pearson correlation coefficient $r_{n,max}$ between the initial number n and $\max(p, n, \pm)$;

f. the mode, the mean, and the standard deviation of the total recurring stopping.

Table 1: Results from the generalised Collatz functions using Java algorithm

$(\mathbf{p}, \mathbf{n}, \pm)$	3n+1	3n-1	5n+1	5n-1	7n+1	7n-1
How many $\mathbf{n}'\mathbf{s}$ of type 1	10000	3244	256	1120	82	13 (only 2^n)
How many $\mathbf{n}'\mathbf{s}$ of type 2	0	6756	470	0	0	0
How many $\mathbf{n}'\mathbf{s}$ of type 3	0	0	9274	8880	9918	9987
Corr. Coeff. $\mathbf{r_{n,\sigma}}$ of $\mathbf{n}$ & $\boldsymbol{\sigma}(\mathbf{p}, \mathbf{n}, \pm)$	0.2054	0.2892	-0.0091	-0.0813	-0.0947	-0.1501
Corr. Coeff. $\mathbf{r_{n,max}}$ of $\mathbf{n}$ & $\mathbf{max}(\mathbf{p}, \mathbf{n}, \pm)$	0.0881	0.1771	0.0711	0.1078	0.0220	0.0029
Mode of $\boldsymbol{\sigma}(\mathbf{p}, \mathbf{n}, \pm)$ (and its frequency)	52 (190)	39 (220)	10 (23)	33 (19)	17, 16 (8)	NA
Mean of $\boldsymbol{\sigma}(\mathbf{p}, \mathbf{n}, \pm)$	84.87	56.28	146.41	160.84	66.83	73.41
Standard deviation of $\boldsymbol{\sigma}(\boldsymbol{p}, \boldsymbol{n}, \pm)$	46.59	30.48	82.64	101.25	30.58	39.16

[2] See Appendix 2 for a recall of the definition.

Chart 1: Plot of $\sigma(3, n, +)$ as a function of n

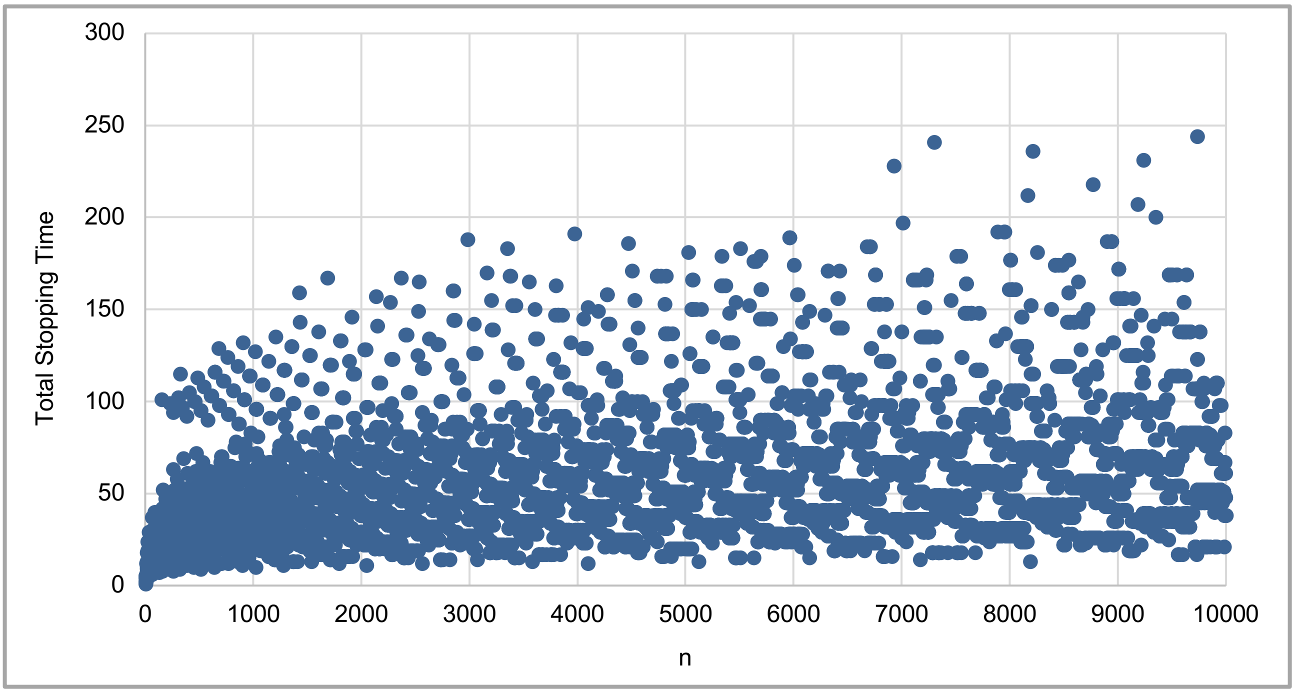
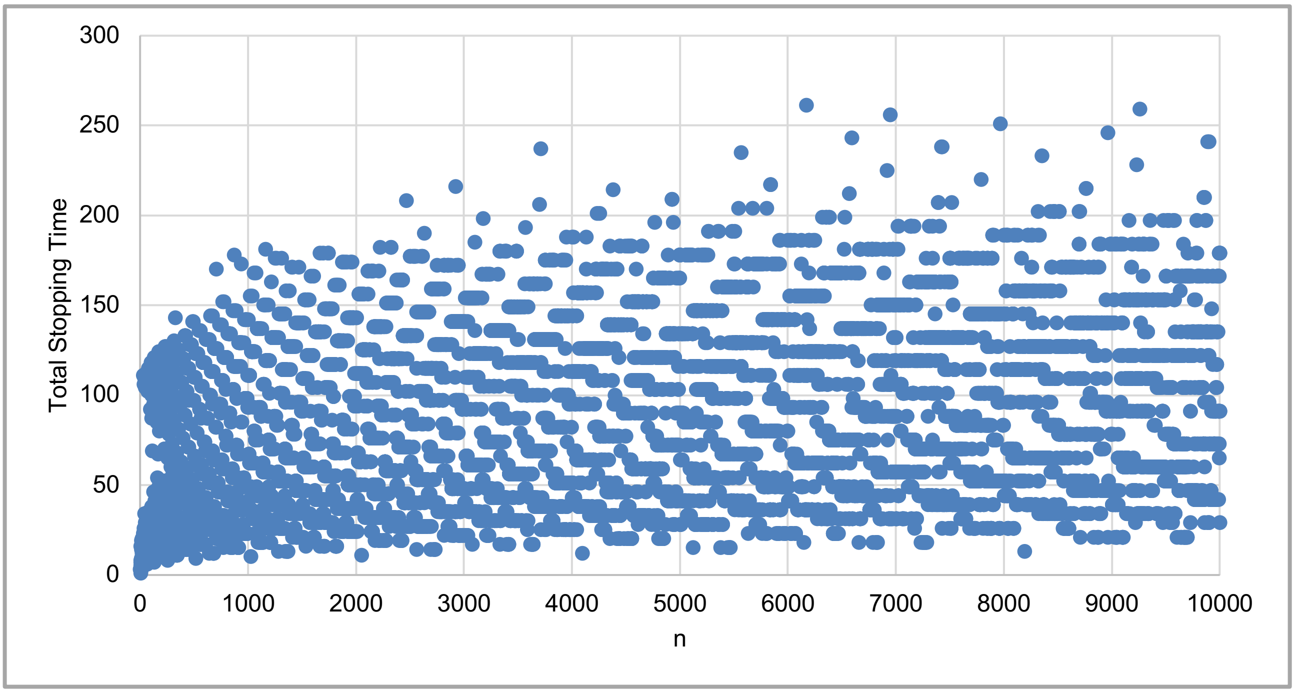


Chart 2: Plot of $\sigma(3, n, -)$ as a function of n



Across the various $(p, n, \pm)$, we tested the hypothesis that the max number or the total stopping time increases as either n becomes larger or p does. However, we found the correlation coefficient to be close to 0 in most cases and no trend either down the list of numbers, or across the values of the prime numbers was found. Hence, it seems that the behavior of the sequence goes randomly up and down, making it hard to solve.

A Real-World Application: Hailstones

How does hail occur?

Let us take a pause from mathematics for a minute and understand the formation of a hailstone, and then we shall come back to formulate the analogy.

A hailstone starts as warm inflow of air is pulled into a storm: the warm air rises and gets caught in an updraft. When this rises above freezing level (3000-4000 m), water molecules begin to freeze. This forms ice which remains in the cloud. Other water molecules interact with the ice, attach themselves, and a chain reaction occurs that leads to larger ice. When the updraft cannot support the weight anymore, the ice falls for what we know as hail (Thomas, n.d.).

Fig 1. Hail (Thomas, n.d.)

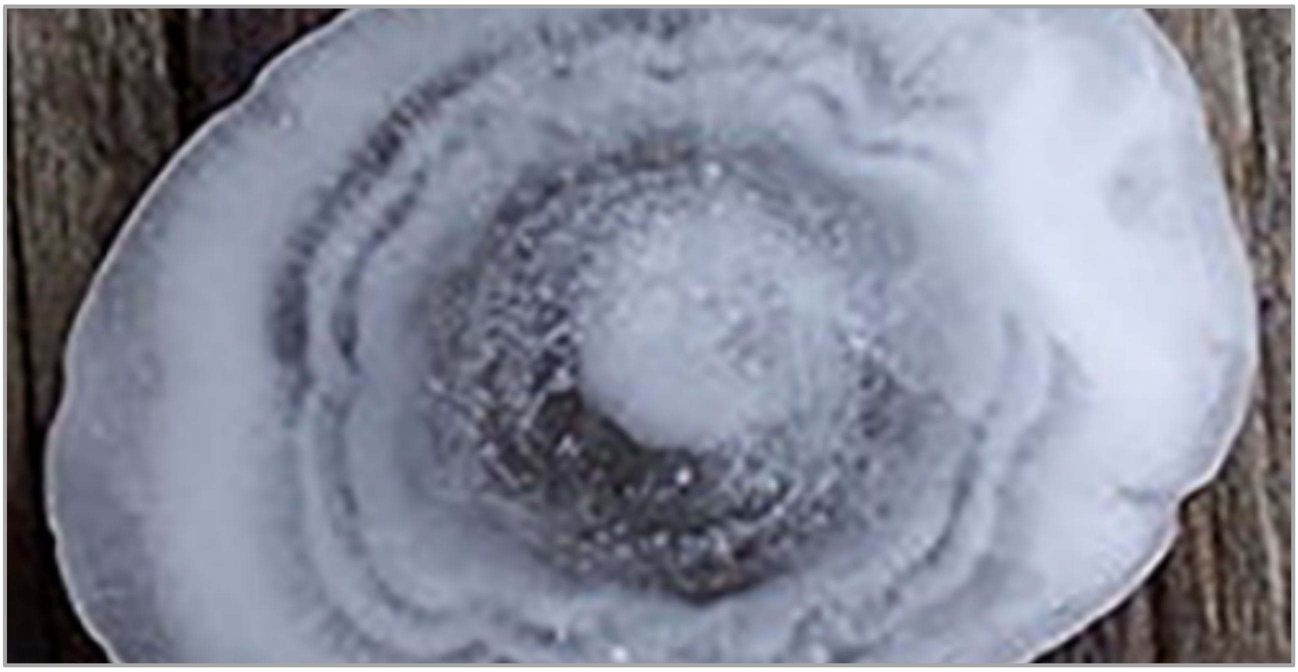
The size of hail is an important consideration in terms of the potential damage it can cause. It can depend on many factors (see analysis later); however, up to 0.5 cm diameter hail happens very frequently and generally does not cause much damage. Warning signs start at the 2 cm size and most damaging hailstones, with highest frequencies are from 4 cm to 8 cm in size (National Weather Service, n.d.).

The largest hailstone ever has even reached 20 cm in size (Allen, et al., 2020). In 2020, more than $20 billion USD in global insurance losses was caused by severe convective storms in USA, with hail as the largest contributor (Allen, et al., 2020). Given hailstones can cause significant damage to property and sometimes even life, it is important and yet very frustrating that our ability to predict the frequency of hailstones and their response to climate variability is limited. This is a bit akin to our ability to predict the maximum value and stopping time for the Collatz sequence.

Just like super computers, advanced data analysis techniques (Harnett, 2019; Tao, 2019) have furthered our ability to assess the Collatz conjecture. Similarly, networks of impact sensors, trained weather observers, data from hail pads, advancements in radar, and satellite technology have significantly improved our understanding of the hailstone phenomenon (Allen, et al., 2020; Kumjian & Lombardo, 2020).

Dry and Wet Growth

Fig 2: Dry and Wet Growth (Thomas, n.d.)



In his paper *The Theory of Hailstone Formation* (1937), Schumann was the first to produce a detailed mathematical theory for hail growth (Schumann, 1937). At that time, the generally accepted theory about the formation of hailstones attributed the process to the capture of super-cooled water drops which lie in the path of the hailstone. More precisely, there was the misconception that only water that can freeze can be accreted (grow), with the surplus part forced to shed. However, in 1959 List discovered spongy ice in hailstones, a form of accretion of super-cooled drops in which the heat transfer is inadequate to freeze all the water, but the excess water is still included within the growing ice. This led to the next theory of hailstone growth (List, 1963), and now the two different types are referred as dry growth (the former) and wet growth (the latter). This is consistent with the change of temperature inside the cloud and the "layered look'' of hails. The temperature of the region the hail is crossing determines the type of growth: the dry one happens when the air temperature is well below the freezing point, making the droplets instantly freeze, which in turns leaves cloudy ice layer. If the temperature is below freezing (<0° C) but not below -30° C, then the phenomenon of wet growth occurs; this type of growth is slow and lets the air bubbles escape giving the clear ice layer.

The Physics Behind the Formation of Hailstones

To run simulations and make comparisons between the model of a real-life hailstone and a Collatz-like hailstone, we need a treatment of hailstone growth from the physical point of view. Although there are three main shapes of hails (conical, irregular and spheroidal, which can be sphere or ellipsoids) (Macklin, 1977) and while the predominant form of large hailstones is triaxial ellipsoid (List, 1963), we assume the all the hailstones that we consider have spherical symmetry: this not only simplifies the computations (Macklin, 1977, p. 74), but it is also consistent with the assumptions made by most papers on the matter. However, according to some papers (Nelson, 1983, p. 3), this leads to an underestimation of the size of hails.

Because of this assumption, we will also accept that the growth of the hailstone is instantaneously homogeneous, and that the density of the hailstone is constant. This is apparently in contrast with the dry and wet growth (Kumjian & Lombardo, 2020). Nevertheless, despite there being a difference of density between the layers (List, 1963) with spongy ice having higher density, such layers are rare and in general the density approximately ranges from $0.88g/cm^3$ to $0.917g/cm^3$ (Macklin, 1977, p. 75). Hence, for computational purposes, the hailstone density may be taken to be equal to $0.9g/cm^3$.

Homogeneity assumptions are made also about the cloud: hail is associated with high, vertical cumulonimbus clouds, the kind of clouds that produce severe thunderstorms. The cloud base is supposed to be $2km$ and the top $10km$, with hailstone embryos of starting radius of $0.30\ cm$, and its density is considered as constant with a value of $0.5g/m^3$ (Schumann, 1937, pp. 3-9) and (Nelson, 1983, pp. 1965-1973).

Moreover, no electrical forces are supposed to be involved and the process is viewed as exclusively mechanical. Also, no mass is lost due to melting, motivated also by the fact that hailstones with radius greater than 0.5cm (emphasized in this study) lose little mass due to melting (Gokhale, 1975). Therefore, the forces which the hailstone is subjected to are:

1. Gravity $F_G = gM_H$ (where M_H is the mass of the hailstone);
2. Drag force $F_d = \frac{1}{2} dV_H^2 K_d A$ where d is the density of the fluid (namely, the cloud), V_H is the velocity of the hailstone with respect to the fluid, A is the cross-sectional area - which is in this case πR_H^2 , being R_H the radius of a spherical hailstone - and K_d is the drag coefficient – a dimensionless number;
3. The force coming from the total momentum imparted to the water from the hailstone, during the perfectly inelastic collision. Consistently with (Macklin, 1977, p. 66), the velocity of the single water droplets with respect to the cloud is assumed to be zero.

Most measurements of the drag coefficients of hailstones (Nelson, 1983) lead to the conclusion that a reasonable value for the drag coefficient of spherical hailstones is about 0.55, without further assumption about the smoothness of hails; indeed it seems they are unaffected by the *drag crisis* (Macklin, 1977, p. 76).

It has been observed and assumed (Macklin, 1977, p. 66) that hailstones grow only when they drop: therefore the equation of the motion is the one for a generalized variable mass system, where no mass is lost: hence, Newton's second law of motion gives us the change of the momentum with respect to the frame of the cloud as compared with equation (10) and (20) in (Biswal, 2021).

$$\overrightarrow{F_d} + \overrightarrow{F_G} = M_H(t) \frac{d\overrightarrow{V_H}(t)}{dt} - (-\overrightarrow{V_H}(t)) \frac{dM_H(t)}{dt}. \qquad (6)$$

Taking as the positive sign the downward one, we have

$$-F_d + F_G = M_H(t) \frac{dV_H(t)}{dt} - V_H(t) \frac{dM_H(t)}{dt}. \qquad (7)$$

Since we are assuming that our hailstones have spherical shape and constant density, the formula relating the mass of the hailstone M_H to its radius R_H is:

$$M_H = \frac{4}{3} \pi R_H^3 \rho, \qquad (8)$$

which implies that the variation is

$$\frac{dM_H}{dt} = 4\pi\rho R_H^2 \frac{dR_H}{dt}. \qquad (9)$$

While traveling, the hailstone “scoops” the water droplets that it meets, and in an instant of time dt the amount of water collected is given by the cylinder of height $V_H\, dt$ and of cross-sectional area πR_H^2. Therefore, we have

$$\frac{\mathrm{d}M_H}{\mathrm{d}t} = 4\pi\rho R_H^2 \frac{\mathrm{d}R_H}{\mathrm{d}t} = \pi R_H^2 V_H d \qquad (10)$$

which implies that the rate of change of the radius in function of the time is

$$\frac{dR_H}{dt} = \frac{V_H d}{4\rho}. \qquad (11)$$

However, we need a formula which correlates the change of the radius to the change of height of the hailstone. To get it, consider the total vertical velocity of the hailstone with respect to the ground: on one hand, it can be expressed as $dh_H\,/\,dt$, and on the other hand it is $u - V_H$, where u is the velocity of the cloud w.r.t. the ground (the updraft speed). Thus, using the chain rule we have

$$\frac{dR_H}{dh_H}\frac{dh_H}{dt} = \frac{V_H d}{4\rho} \quad \Rightarrow \quad \frac{dR_H}{dh_H}(u - V_H) = \frac{V_H d}{4\rho} \quad \Rightarrow \quad \frac{dR_H}{dh_H} = \frac{V_H d}{4\rho(u - V_H)}. \qquad (12)$$

Despite we could use a better approximation for the velocity of hailstone V_H, coming from equation *(7)*, we will use the fall-speed derived from the balance between gravity and the drag force for spherical particles:

$$V_H = \left(\frac{8g\rho R_H}{3K_d d}\right)^{\frac{1}{2}}. \qquad (13)$$

Denote by C the constant $\left(\frac{8g\rho}{3K_d d}\right)^{\frac{1}{2}}$ in equation *(13)*, we can rewrite equation *(12)* as

$$\left(\frac{u}{C\sqrt{R_H}} - 1\right)\mathrm{d}R_H = \frac{d}{4\rho}\, dh_H. \qquad (14)$$

Then we can integrate the previous ordinary differential equation, obtaining

$$R_{H,fin} = \frac{u}{C} + \left(\left(\frac{u}{C}\right)^2 + R_{H,in} + \frac{d}{4\rho}(h_{in} - h_{fin}) - \frac{2u}{C}\sqrt{R_{H,in}}\right)^{1/2}. \qquad (15)$$

For clarity, we summarise the notation used in the paper:

R_H : radius of the hailstone
h_H : height of the hailstone
h_0 : initial height of the hailstone
V_H : (vertical) speed of the hailstone with respect to the water droplets inside the cloud
M_H : mass of the hailstone
ρ : density of the hailstone
d : density of the cloud
u : updraft velocity of the wind (see Table 11 in Appendix 3 for how the updraft velocity changes with hailstone size)
g : gravity constant (equal to $9.8 m\,/\,s^2$)
K_d : drag coefficient

The analogy with the conjecture

Given this brief introduction to hailstones, let us introduce how we want to measure the similarity between the meteorological phenomenon and the mathematical conjecture. We need to compare two frameworks.

Collatz-like hailstones

We assume that the behavior of the trajectory of the hailstone can be related to a Collatz-like function. Rigorously speaking, this means we are assuming that there exists a strictly increasing real-valued function $F: \mathbb{R} \to \mathbb{R}$ such that there exists a sequence of "meaningful times" $\{t_k\}_{\{k \geq 0\}}$ (where $t_0 = 0$) such that the height $h(t)$ of the hailstone at time $t = t_k > 0$ can be expressed as

$$h(t_k) \coloneqq h_k = F\left(f^{(k)}(\lfloor F^{-1}(h(0)) \rfloor) \right) \qquad (16)$$

where the floor function $\lfloor x \rfloor$ of a real number x is the greatest integer less or equal than x. The real functions that we will try in place of F will be square-root, linear, quadratic, logarithmic and exponential like. The sequence of "meaningful times" corresponds to the instances when the motion of the hailstone changes direction in the process of going up and down.

Real hailstones – Nelson's model

Ideally speaking we could consider the actual trajectories of real hailstones, but we understand them up to a certain extent. We do not have a formula that describes accurately the position $p(t)$ of the hailstone at any time t (notice that $p(t)$ is a 3-dimensional vector $(x(t), y(t), h(t))$ where $(x(t), y(t))$ are the horizontal components). However, we do have some formulae about the growth of the radius of the hailstone (see equation *(15)*), and we have some papers with models and simulated trajectories.

We would compare the height of the hailstone in real life and the height of the hailstone predicted by the Collatz-like behavior and see whether the difference is consistently small. However, given the highly unpredictable nature of hailstones and the lack of a math formula, this is not yet doable: we compare instead we think we may have a better chance of comparing the radii that we would obtain from the two different models and comparing those model output values with the sizes that we have from direct observation.

The table below (Table 2) shows the number of model grid points that produce hail of a given diameter as a function of initial height (Nelson, 1983, p. 1973). One limitation of the study is that there are very few real observed tabulated data of the final diameter (or radius) of the hailstone given the starting height. However, Table 2 is the ideal tabulation of data, and the study can be used, in future, to compare to other observed data of similar nature as well.

Table 2: Nelson's model – Final hailstone diameter (cm) as a function of initial height, h_0

h_0 (km)	Final hailstone diameter (cm)									
	1.0	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**Total**	**%**
≤ 5.0	19	-	-	-	-	-	-	-	19	9.1
5.0-6.0	29	8	1	-	-	-	-	-	38	18.1
6.0-7.0	27	11	7	2	1	1	-	-	49	23.3
7.0-8.0	23	9	3	6	1	1	-	1	44	21.0
8.0-9.0	15	9	6	2	-	-	-	1	33	15.7
9.0-10.0	14	8	1	3	-	-	-	1	27	12.9
Total	127	45	18	13	2	2	0	3	210	
Total %	60.5	21.4	8.6	6.2	1.0	1.0	0.0	1.4		

Study Method

To analyze the suitability of the Collatz conjecture to the observed values of hailstone formation, we used equation *(16).* For simplicity we will assume that h_0 is an integer. We will test three types of equations:

a. Polynomial $F(h) = (h - h_{\text{inf}})^{\alpha} + 1$, with the exponent α ranging from [0.5, 2.0] (i.e., from square root to square). We shall try values such as 0.5, 1.0, 2.0 (3 cases)

b. Logarithmic $F(h) = ln(h - h_{\text{inf}}) + 1$ (1 case for log natural, although a quick assessment showed that $\log_{10}$ does not make a material difference)

c. Exponential $F(h) = exp(h - h_{\text{inf}}) + 1$ (1 case)

As will be clear later, unit of measure is a key variable in equation *(16).* We shall try units ranging from km to mm, although cm is the default unit (since radius is expressed in cm). All the above would be tried for starting heights ranging from the freezing point, namely 4 km, to 10 km, in increments of 500m to test viability and in increments of 100m for viable solutions.

This gives rise to a maximum of types of cases [5] x [7] UoM (from km to mm), equalling 35 scenarios. Each scenario would be tried for 13 values (4.0 km to 10.0 km in increments of 500m) and for 61 values for viable solutions only (4.0 km to 10.0 km in increments of 100m) depending on the starting height. This would then be compared to the real observed values in nature (Nelson, 1983) and we will use goodness-of-fit to estimate the best possible scenario and equation. We assess, as feasible, only those functions where the following criteria are met:

1. Most (>50%) of the estimated radii values are in the "believable" category for hailstone radii i.e., less than 10 cm;
2. There is reasonable dispersion in the range of estimated radii to analyze goodness-of-fit (some values between 0.5 cm and 2.5 cm and range should be at least 1.0 cm;

3. The estimated radii should have enough points of inflection (i.e., rises and drops like a Collatz Conjecture or observed hailstone radii) and not be a monotonic function. For instance, if the estimated values of R_h for different values of h, steadily increase (or decrease) as h increases (or decreases), then this would not be analogous to how the values of $f(n)$ fluctuate (which increase and decrease multiple times before converging to 1) for different values of n. Hence, we would not consider such a function to behave analogously to a Collatz function and would not consider them as feasible

Table 3: Summarising the assumptions for values for the variables used in the computation

Variable	Definition	Unit	Value
$\boldsymbol{\rho}$	Hailstone density	g/ cm^3	0.9
$\boldsymbol{g}$	Gravity constant	cm /s^2	980
$\boldsymbol{R_0}$	Initial radius	cm	0.30
$\boldsymbol{K_d}$	Drag coefficient	no unit	0.55
$\boldsymbol{d}$	Density of cloud	g/ cm^3	0.0000005
$\boldsymbol{u}$	updraft speed	m /s	25
$\boldsymbol{h_{inf}}$	Bottom height	cm	200000
$\boldsymbol{C}$	Constant	$m^{1/2}$/s	924.80

Analysis

If we consider the sequence of heights $\{h_k\}_{k\geq 0}$ corresponding to the Collatz sequence via the conversion formula, then

- if $h_{k+1} \geq h_k$ (hailstone rises in height) then there is no change in the radius. In particular $R_{k+1} = R_k$;
- if $h_{k+1} \leq h_k$ (hailstone drops in height) then the radius changes according to equation 15. In particular $R_{k+1} > R_k$.

Then the final radius R_{final} is achieved when h_{final} is equal to 2000 meters, which corresponds when $F(h_{final}) = 1$ that is when the Collatz sequence reaches 1. We computed the final radii for given starting heights h_0 and the conversion formulas above reported, and we summarize the results in the tables that follow. The first column contains the initial height (the height of the embryo of the hailstone), while the others contain the final radii expressed in cm. For the initial height, we consider values from 4 km (or equivalent, depending on the unit of measure) to 10 km (or equivalent). This is because our observed comparison data is Table 2 (Nelson, 1983) and that data ranges from ≤ 5.0 km to 10.0 km.

We changed the updraft speed from 25 m/s to other values between 20 m/s to 50 m/s but there was no material difference in the estimation of the radii. For instance, simulating the radii for the linear equation with α = 1.0 and unit of measure being cm, the change was less than 0.01 cm; it was found to be similar for other scenarios too. Hence, a constant updraft speed of 25 m/s was assumed.

Scenario 1: Polynomial equation for α = 0.5

Table 4: Values of the final radius (cm) with α = 0.5

$h_0\ (m)$	Km	Hm	Dm	m	dm	cm	mm
4000	0.30	0.30	0.57	9.35	All values getting increasingly larger and significantly greater than 10.0 cm (typically greater than 10000 cm) as unit of measure gets smaller; hence not assessed		
4500	0.30	0.30	31.71	8.49			
5000	0.30	32.83	0.44	40.79			
5500	0.30	0.30	38.01	20.81			
6000	0.30	32.83	0.61	59.62			
6500	0.30	0.31	6.79	26.18			
7000	0.30	0.31	33.14	1,032.65			
7500	0.30	0.30	6.46	132.50			
8000	0.30	0.31	7.21	68.55			
8500	0.30	0.30	35.19	92.36			
9000	0.30	32.82	33.21	164.32			
9500	0.30	0.30	277.36	32,162.28			
10000	0.30	0.32	7.97	143.77			

Considering the results, we can see that no case respects all the viability criteria. Indeed, for Km, there is virtually no change in the radius (0.30 cm) and hence no variation that we can use goodness-of-fit to assess against Table 2. For Hm, of the 13 values assessed, 10 of them showed virtually no variation (between 0.30 and 0.31 cm) and 3 values that did show a variation, were very large (>32 cm) to be considered believable. For Dm, there were only 3 values (out of 13 estimated) that were all in a narrow range (<1.0 cm), 4 values that were large and yet believable (5.0 cm < r < 10.0 cm), and 7 values that were very large (>10.0 cm). For m, only 2 values were <10.0 cm and 11 values unbelievably large (>10.0 cm). For dm, cm, mm, all values were greater that 10.0 cm.

Hence, as we can see, Scenario 1 did not yield any feasible options that meet the three criteria of: believability of size, reasonable dispersion, sufficient points of inflection.

Scenario 2: Polynomial equation for α = 1.0

Table 5: Values of the final radius (cm) with α = 1.0

h_0(m)	Km	Hm	Dm	m	dm	cm	mm
4000	0.30	0.30	0.30	0.30	0.35	0.48	11.54
4500	0.30	0.30	0.30	0.30	0.50	1.84	4.86
5000	0.30	0.31	0.30	0.30	0.35	1.43	3.35
5500	0.30	0.30	0.31	0.30	0.36	0.62	21.17
6000	0.30	0.31	0.30	0.31	0.36	1.56	4.62
6500	0.30	0.30	0.30	0.30	0.47	0.70	5.99
7000	0.30	0.30	0.31	0.34	0.45	1.52	4.63
7500	0.30	0.30	0.30	0.31	0.43	0.66	12.65
8000	0.30	0.30	0.30	0.30	0.49	0.90	7.89
8500	0.30	0.30	0.31	0.31	0.61	0.88	16.31
9000	0.30	0.31	0.31	0.31	0.35	1.34	31.99
9500	0.30	0.30	0.32	0.56	0.45	1.51	60.02
10000	0.30	0.30	0.30	0.31	0.52	1.15	10.41

Considering the results, we can see that only one case respects all the viability criteria. Indeed, for Km, Hm, Dm, m, there is virtually no change in the radius (all values between 0.30 cm and 0.34 cm) and hence no variation that we can use goodness-of-fit to assess against Table 2. For dm, too, all values are <1.0 cm and again, the range is very narrow. For cm, we notice that all the values are believable (<10.0 cm), there is a good range of values between 0.48 cm and 1.84 cm, and the values are not monotonic; they tend to increase and decrease with 9 points of inflection. For mm, only 6 values were believable (<10.0 cm) while 7 values were very large (>10.0 cm)

Hence, as we can see, Scenario 2 yields one feasible option (unit of measure as cm) that meets the three criteria of: believability of size, reasonable dispersion, sufficient points of inflection.

Scenario 3: Polynomial equation for α = 2.0

Table 6: Values of the final radius (cm) with α = 2.0

h_0 (m)	Km	Hm	Dm	m	dm	cm	mm
4000	0.30	0.30	0.30	0.30	0.30	0.33	0.59
4500	0.30	0.30	0.30	0.30	0.30	0.34	0.66
5000	0.30	0.30	0.30	0.30	0.30	0.34	0.73
5500	0.30	0.30	0.30	0.30	0.31	0.35	0.81
6000	0.30	0.30	0.30	0.30	0.31	0.36	0.88
6500	0.30	0.30	0.30	0.30	0.31	0.37	0.95
7000	0.30	0.30	0.30	0.30	0.31	0.37	1.02
7500	0.30	0.30	0.30	0.30	0.31	0.38	1.09
8000	0.30	0.30	0.30	0.30	0.31	0.39	1.16
8500	0.30	0.30	0.30	0.30	0.31	0.39	1.24
9000	0.30	0.30	0.30	0.30	0.31	0.40	1.31
9500	0.30	0.30	0.30	0.30	0.31	0.41	1.38
10000	0.30	0.30	0.30	0.30	0.31	0.42	1.45

Considering the results, we can see that no case respects all the viability criteria. Indeed, for Km, Hm, Dm, m, dm, cm there is virtually no change in the radius (all values between 0.30 cm and 0.42 cm) and hence no variation that we can use goodness-of-fit to assess against Table 2. For mm, the values are all believable (<1.5 cm) and there is some variation (ranging from 0.59 cm to 1.45 cm). However, the values monotonically increase with height unlike how the Collatz function or observed formation of hail in a hailstone behave. Hence, we do not consider this option as feasible either.

Hence, as we can see, Scenario 3 yields no feasible option that meets the three criteria of: believability of size, reasonable dispersion, sufficient points of inflection.

Scenario 4: Natural logarithm (ln) equation

Table 7: Values of the final radius (cm) with ln-equation

<table>
<tr><th>h_0 (m)</th><th>Km</th><th>Hm</th><th>Dm</th><th>m</th><th>dm</th><th>cm</th><th>mm</th></tr>
<tr><td>4000</td><td>0.77</td><td>3.18E+20</td><td colspan="5" rowspan="13">These cases are not explicitly reported since the final value exceeds 10.0 cm; in fact, most values exceed 1.0E+10 cm</td></tr>
<tr><td>4500</td><td>0.77</td><td>1.20E+10</td></tr>
<tr><td>5000</td><td>0.30</td><td>NA</td></tr>
<tr><td>5500</td><td>0.30</td><td>1.96E+14</td></tr>
<tr><td>6000</td><td>0.77</td><td>NA</td></tr>
<tr><td>6500</td><td>0.77</td><td>1.57E+62</td></tr>
<tr><td>7000</td><td>0.77</td><td>2.91E+93</td></tr>
<tr><td>7500</td><td>0.77</td><td>1.96E+15</td></tr>
<tr><td>8000</td><td>1.90E+14</td><td>4.16E+72</td></tr>
<tr><td>8500</td><td>1.90E+14</td><td>1.37E+36</td></tr>
<tr><td>9000</td><td>0.30</td><td>NA</td></tr>
<tr><td>9500</td><td>0.30</td><td>8.43E+30</td></tr>
<tr><td>10000</td><td>1.96E+14</td><td>4.75E+98</td></tr>
</table>

Considering the results, we can see that no case respects all the viability criteria. For Km, we have 10 values that are within a very narrow range (between 0.3 and 0.8 cm) and 3 values that are unbelievably large (> 1.9E+14). For Hm, 10 values are unbelievably large (>1.2E+10) while 3 values are so large that they could not be computed in Excel. For all other units of measure, the values are larger that 1.0E+10 or could not be computed.

Hence, as we can see, Scenario 4 yields no feasible option primarily because the values are unbelievably large.

Scenario 5: Exponential equation

Table 8: Values of the final radius (cm) with exp-equation

h_0 (m)	Km	Hm	Dm	m	dm	cm	mm
4000	0.30	0.30	0.30	0.30	0.30	0.33	0.59
4500	0.30	0.30	0.30	0.30	0.30	0.34	0.66
5000	0.30	0.30	0.30	0.30	0.30	0.34	0.73
5500	0.30	0.30	0.30	0.30	0.31	0.35	0.81
6000	0.30	0.30	0.30	0.30	0.31	0.36	0.88
6500	0.30	0.30	0.30	0.30	0.31	0.37	0.95
7000	0.30	0.30	0.30	0.30	0.31	0.37	1.02
7500	0.30	0.30	0.30	0.30	0.31	0.38	1.09
8000	0.30	0.30	0.30	0.30	0.31	0.39	1.16
8500	0.30	0.30	0.30	0.30	0.31	0.39	1.23
9000	0.30	0.30	0.30	0.30	0.31	0.40	1.30
9500	0.30	0.30	0.30	0.30	0.31	0.41	1.38
10000	0.30	0.30	0.30	0.30	0.31	0.42	1.45

Considering the results, we can see that no case respects all the viability criteria. Indeed, for Km, Hm, Dm, m, dm, cm there is virtually no change in the radius (all values between 0.30 cm and 0.42 cm) and hence no variation that we can use goodness-of-fit to assess against Table 2. For mm, the values are all believable (<1.5 cm) and there is some variation (ranging from 0.59 cm to 1.45 cm). However, the values monotonically increase with height unlike how the Collatz function or observed formation of hail in a hailstone behave. Hence, we do not consider this option as feasible either.

Hence, as we can see, Scenario 5 yields no feasible option that meets the three criteria of: believability of size, reasonable dispersion, sufficient points of inflection.

Therefore, summarizing our analysis across the 5 scenarios assesses, the only viable solution that meets all three criteria is the linear equation ($\alpha = 1$) and with centimeters as unit of measure.

Now we proceed to address the second question in this case: how suitable the description is. To assess the suitability of the function using goodness-of-fit, we further estimated the radii in increments of 100m and obtained the values in Table 9 below.

Table 9: Estimated radius (and diameter) in cm for different heights (in km) and increments of 100m each

Height (km)	Radius (cm)	Diameter (cm)	Height (km)	Radius (cm)	Diameter (cm)	Height (km)	Radius (cm)	Diameter (cm)
4.0	0.48	0.96	5.1	0.53	1.07	6.1	0.64	1.28
4.1	0.65	1.29	5.2	0.70	1.41	6.2	20.39	40.79
4.2	3.36	6.71	5.3	0.59	1.18	6.3	0.60	1.21
4.3	0.46	0.91	5.4	0.92	1.83	6.4	0.73	1.47
4.4	0.60	1.20	5.5	0.62	1.24	6.5	0.70	1.41
4.5	1.84	3.68	5.6	0.64	1.27	6.6	108.78	217.57
4.6	0.64	1.28	5.7	34.20	68.39	6.7	0.67	1.33
4.7	0.52	1.04	5.8	1.23	2.46	6.8	0.79	1.58
4.8	0.74	1.47	5.9	0.60	1.20	6.9	0.68	1.36
4.9	1.02	2.04	6.0	1.56	3.12	7.0	1.52	3.03
5.0	1.43	2.86						
7.1	0.91	1.82	8.1	0.94	1.89	9.1	0.79	1.57
7.2	0.77	1.53	8.2	1.77	3.53	9.2	1.51	3.03
7.3	2.09	4.17	8.3	0.86	1.72	9.3	1.05	2.10
7.4	1.29	2.57	8.4	1.00	1.99	9.4	1.80	3.61
7.5	0.66	1.32	8.5	0.88	1.76	9.5	1.51	3.01
7.6	20.57	41.14	8.6	1.71	3.43	9.6	1.39	2.77
7.7	1.14	2.28	8.7	0.86	1.72	9.7	1.06	2.11
7.8	1.37	2.73	8.8	1.13	2.25	9.8	6.10	12.21
7.9	1.22	2.43	8.9	1.40	2.80	9.9	1.78	3.55
8.0	0.90	1.79	9.0	1.34	2.67	10.0	1.15	2.31

We then summarised the values in Table 9 by tabulating the rows in km, in increments of 1.0 km and grouping diameters for <1.0 cm, 1.0-1.49 cm, 1.5-1.99 cm, 2.0-2.49 cm, 2.5-2.99 cm, 3.0-3.49 cm, 3.5-3.99 cm, 4.0-4.49 cm, 4.5 cm and above. We compared Table 10 to Table 2 (Nelson, 1983) and estimated feasibility of our hypothesis using goodness-of-fit.

Table 10: Scenario 2 model - final hailstone diameter (cm) as a function of h_0

h_0 (km)	Final hailstone diameter (cm)									
	≤1.0	1.5	2.0	2.5	3.0	3.5	4.0	≥4.5	Total	%
≤5.0	2	5	-	1	1	-	1	1	11	18.0%
.45.0-6.0	-	6	1	1	-	1	-	1	10	16.4%
6.0-7.0	-	6	1	-	-	1	-	2	10	16.4%
7.0-8.0	-	1	3	2	2	-	-	2	10	16.4%
8.0-9.0	-	-	5	1	2	1	1	-	10	16.4%
9.0-10.0	-	-	1	3	1	2	2	1	10	16.4%
Total	2	18	11	8	6	5	4	7	61	
Total %	3.3%	29.5%	18.0%	13.1%	9.8%	8.2%	6.6%	11.5%		

For the values obtained in Table 10, when compared to Table 2, we get a $\chi^2 = 0.70$.

Chart 3: Observed vs estimated frequencies by diameter (absolute)

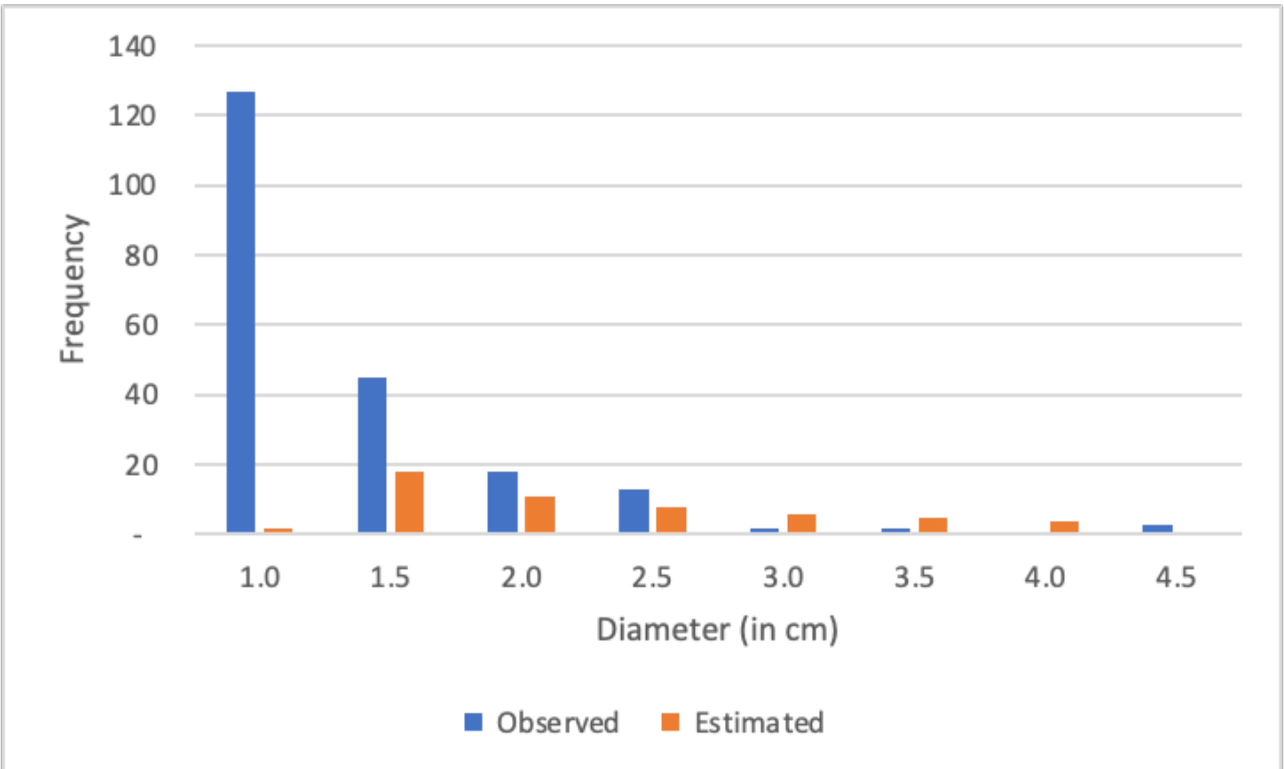
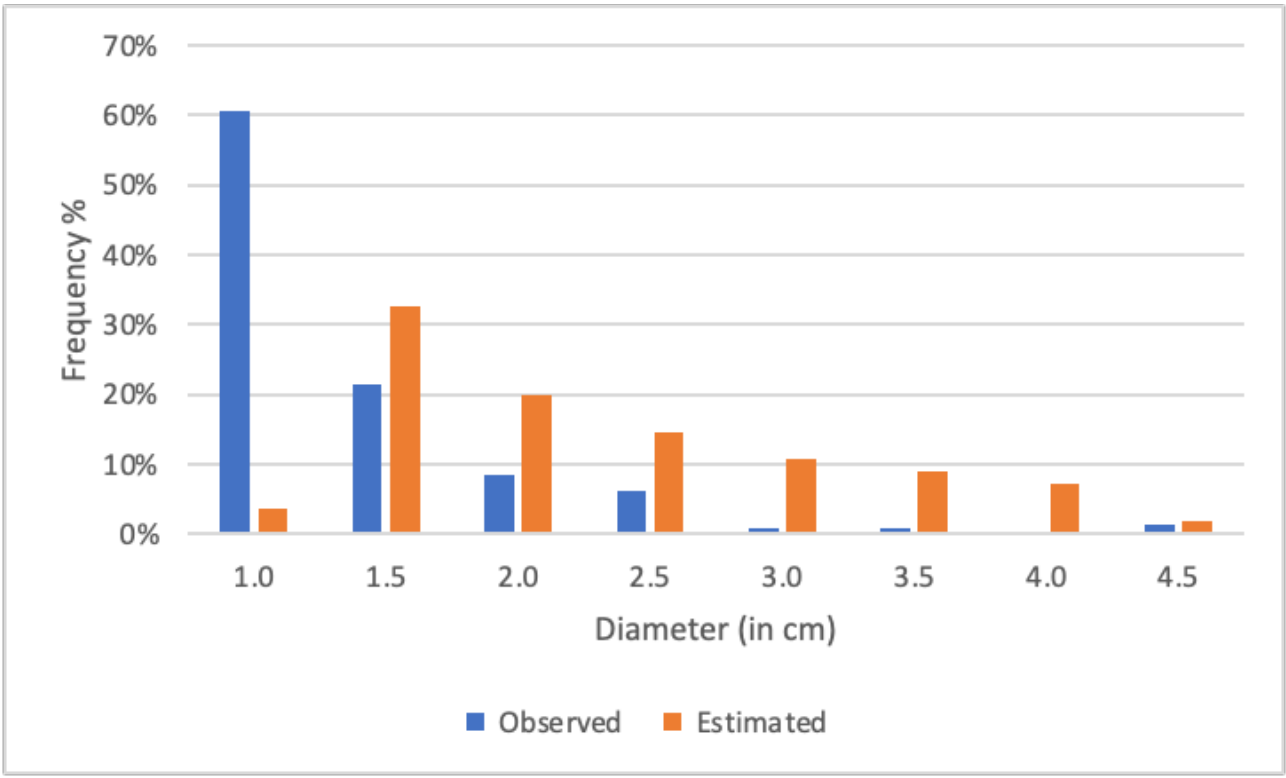


Chart 4: Observed vs estimated frequencies by diameter (%)



Conclusion

If χ^2 were >1.0, we could have rejected the hypothesis that there is a feasible analogy between the formation of hail in a hailstone and the Collatz conjecture. But since the χ^2 is <1.0 (0.70), we cannot reject the hypothesis and there is a possibility that the radii in the formation of hail in a hailstone could mirror the Collatz function, for a linear equation with cm as the unit of measure and all the other assumptions there.

The paper has certain limitations, in terms of the assumptions we have made. Some of the potential next steps worth evaluating would be assuming:

- different shapes for the hailstones (other than spherical)
- different more generalized Collatz functions of p, sign from equation 5 (other than 3n+1)
- different updraft speeds (from 20 m/s to 50 m/s)
- other forms of the equation, other than linear, logarithmic, or exponential
- values of α, other than 0.5, 1.0 and 2.0
- working with other values of observed data

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Appendices

Appendix 1: Java Code

Here we add a brief discussion about the algorithm used to test the Collatz-like functions: for the algorithm itself, see GitHub (https://github.com/rheaaguptaa/collatz-Conjecture). The program takes 4 inputs and generates 4 outputs.

The inputs are the following:

- Range of numbers you want to test the program for $(n1, n2)$
- Prime number (p)
- Sign $(+$ or $-)$

For each number $n_1 \leq n \leq n_2$, and for the Collatz function f $(n/2$ if even or $pn \pm 1$ if odd), it executes f and saves the resultant value in an array. It checks if the resultant value is either 1 or has already been saved in the array earlier (in which case the array would start duplicating). If either of the conditions are met, the program exits the loop and produces the outputs; else it keeps iterating function f.

There is a limitation that if the resultant number reaches or exceeds 2147483647, then we conclude that there is no convergence or duplication; this is a Java programming limitation.

Finally, the algorithm produces as outputs:

- Number (n) for which the calculation is done
- The maximum number it reaches in the successive iterations of f
- The total stopping time to reach 1 or to a duplicate number
- The converging number (either 1, or the duplicate number, or "no convergence" should the formula exceed 2147483647).

Appendix 2: Definition of Pearson correlation coefficient (r) and Chi-square (χ^2)

Correlation coefficient (Glen, n.d.) is used to determine how strong a relationship is between data. It is a value between -1 and +1, where:

- +1 indicates a strong positive relationship
- -1 indicates a strong negative relationship
- 0 indicates no relationship at all.

The most common measure of correlation in statistics is the Pearson Product Moment Correlation (PPMC) or

simply known as the Pearson correlation coefficient (r). It shows the linear relationship between two sets of data $\{x_i\}$ and $\{y_i\}$. It is expressed by the formula

$$r_{xy} = \frac{n\sum_{i=1}^{n} x_i\, y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{[n\sum_{i=1}^{n} x_i{}^2 - (\sum_{i=1}^{n} x_i)\text{^}2\ ]^{1/2}[n\sum_{i=1}^{n} y_i{}^2 - (\sum_{i=1}^{n} y_i)\text{^}2\ ]^{1/2}}. \qquad (17)$$

A chi-square statistic (Glen, n.d.) is a way to show a relationship between two variables. It is a single number that tells you how much difference exists between your observed counts and the counts you would expect if there were no relationship at all in the population. A low value for chi-square means there is a high correlation between the two sets of data. It can be calculated using the formula

$$\chi^2 = \sum_{\mathrm{i}} \frac{(O_i - E_i)^2}{E_i} \qquad (18)$$

where O_i is the observed frequency and E_i is the expected one.

Appendix 3: Relationship between hailstone size and updraft speed (National Weather Service, n.d., p. 3)

Table 11: Hailstone size at different updraft speeds

Hailstone size	Measurement in cm	Updraft speed in m/s
Bb	<0.64	<39
Pea	0.64	39
Marble	1.3	56
Dime	1.8	61
Penny	1.9	64
Nickel	2.2	74
Quarter	2.5	79
Half dollar	3.2	87
Walnut	3.8	97
Golf ball	4.4	103
Hen egg	5.1	111
Tennis ball	6.4	124
Baseball	7.0	130
Teacup	7.6	135
Grapefruit	10.1	158
Softball	11.4	166

Research Scholar Evaluation

Please evaluate the following in detail

1. **Describe some of the scholar's strengths:**

The student is strong motivated and research oriented: indeed, after a session of brainstorming about possible topics, Rhea did some research on her own and proposed herself the Collatz Conjecture as specific research topic. Also, she has already a good sense of what is hard and what is doable in research. Indeed, she didn't hesitate to address questions like "how to make progresses in such hard research problem". Moreover, she has a good sense of time and schedule, pointing out concerns about deadlines and changing her writing/project goals accordingly. Besides this, Rhea knew already JavaScript language and how to use Excel and modeled the project so that she could use her skills at best.

2. **Describe some areas in which the student can improve:**

Writing skills in math (I mean writing formulas in words/latex).

3. **Would you recommend the scholar to a college/university admissions officer? Please describe:**

Absolutely! The student showed a lot of potential and skills that people struggle to have also after having started a research path (please see scholar's strength section). I think Rhea gave an example of how to be grounded to Earth while shooting for the stars (which I think it is one of the best way to start research).

Date: 09/17/2022

Evaluator Name: Morena Porzio

Evaluator Signature: Morena Porzio

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Journal of Student Research

An Estimation of Hailstone Radii using the Collatz Conjecture

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The Shri Ram School Aravali

Morena Porzio
Columbia University

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PDF

ABSTRACT

There is an alleged connection between a much researched, yet unsolved math problem, the Collatz conjecture, and a quite common, damaging meteorological phenomenon: hailstorm. The relationship would be that, in sequences generated by Collatz algorithm, the way in which numbers rise and fall resembles hailstones going up and down inside a cloud, whence the name "Hailstone sequences". The aim of this paper is two-fold: first, to use JavaScript to research on the Collatz conjecture with the perspective of a high-school student. Our algorithm tested a generalized form of the conjecture for multiple primes (3, 5, 7) and signs (+,). The Pearson correlation coefficient found between the initial value and, respectively, the total stopping time or the maximum value reached excluded any linear correlation. The second (and main) goal was to assess the hypothesis whether hailstones could indeed follow a Collatz-like function trajectory, studying the implication on the radii of them. Introducing the concept of conversion *formula*, we estimated the final radii for different functions (straight line, square-root, square, logarithmic, exponential), unit of measures (from Km to mm), and starting heights ranging from 4000m to 10000m, should the motion of hailstones behave like a Collatz function. In all but one case, we did not get radii believable in size, and *reasonably* randomly distributed. For the linear formula (in cm), the -test values between our estimated values and Nelson's model values are above the critical values. Hence, we should reject the initial hypothesis.

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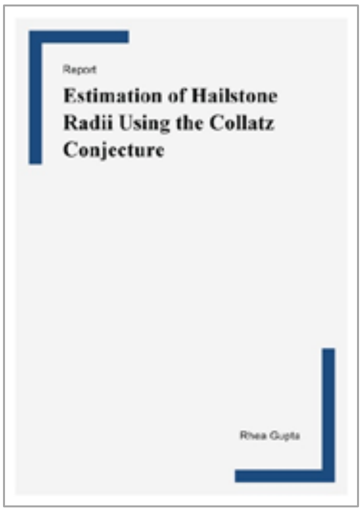

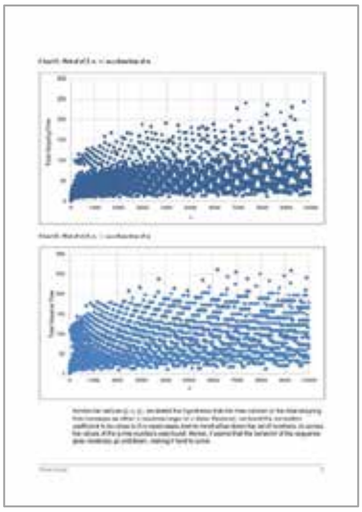

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